

# A Solution to the Gating Paradox.

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**ABSTRACT** We introduce a Markov model for the gating of membrane channels. The model features a possible solution to the so-called gating current paradox, namely that the bell-shaped curve that describes the voltage dependence of the kinetics is broader than expected from, and shifted relative to, the sigmoidal curve that describes the voltage dependence of the activation. The model also predicts some temperature dependence of this shift, but presence of the latter has not been tested experimentally so far.

## INTRODUCTION

The gating of membrane channels is of vital importance for the electrophysiological activity of nerve, heart and muscle. While some of these channels appear to have fractal-like gating (Liebovitch, 1995), most membrane channels do display activity that can be well approximated by a simple Markov process (Korn and Horn, 1988). However, Clay *et al.* (1995) revealed a gating current paradox that has been difficult to explain with a standard type (Hille, 1992) Markov model. The paradox is that the bell-shaped curve that describes the voltage dependence of the kinetics is shifted significantly relative to the sigmoidal curve that describes the voltage dependence of the activation. The standard type models (Hille, 1992) does not allow such a shift. Also the former curve is broader than the one predicted by the standard model.

Here we introduce a new Markov model, that extends and generalizes the standard one. Our generalization consists of introducing an alternative route between the open and the closed positions of the gate. With two routes, or two membrane protein folding pathways, we are able to obtain results consistent with the observed ones. Thus such a model presents a possible resolution of the above paradox. A more complete resolution requires investigation of the detailed physical mechanism present in real membrane channels to see how they compare with the model. The idea with two routes, a rapid one and a slow one, is that the probability of choosing one or the other also depends upon the voltage through a Boltzmann factor. This will affect the kinetics, but not the equilibrium distribution (stationary state), and a relative shift of curves can take place.

## THE MODEL

We imagine that a membrane channel has one open and one closed state as in the simplest standard (Hille, 1992) Markov model for this problem. However, between these states we now assume that there exist two routes ( $i = 1, 2$ ). This gives,

$$C \xrightleftharpoons[\beta_2]{\alpha_1, \alpha_2} O, \quad (1)$$

where the rate constants  $\alpha_1$ ,  $\alpha_2$  and  $\beta_1$ ,  $\beta_2$ , which are functions of voltage (but are constant at any given voltage), control the transitions between the closed (C) and the open (O) states of the gate. The  $\alpha_i$  is the rate for a closed channel to open, and  $\beta_i$  the rate for an open channel to close. We introduce effective rate constants  $\alpha$  and  $\beta$ ,

$$\alpha = p_1\alpha_1 + p_2\alpha_2 \quad (2)$$

$$\beta = p_1\beta_1 + p_2\beta_2 , \quad (3)$$

where the probabilities  $p_1$  and  $p_2$  are related in a standard way to the difference  $\Delta G_b$  in energy barriers that must be overcome for each of the two routes,

$$p_1 = \frac{\exp(-\frac{\Delta G_b}{2kT})}{\exp(\frac{\Delta G_b}{2kT}) + \exp(-\frac{\Delta G_b}{2kT})} \quad (4)$$

$$p_2 = \frac{\exp(\frac{\Delta G_b}{2kT})}{\exp(-\frac{\Delta G_b}{2kT}) + \exp(\frac{\Delta G_b}{2kT})} . \quad (5)$$

Let  $x$  denote the average fraction of gates that are open, or, equivalently, the probability that a given gate will be open, and let us imagine that a Markov (1906) model is suitable to describe the gating. One then has, as usual

$$\frac{dx}{dt} = \alpha(1-x) - \beta x = \frac{x_\infty - x}{\tau} , \quad (6)$$

where

$$x_\infty = \frac{\alpha}{\alpha + \beta} , \quad (7)$$

$$\tau = \frac{1}{\alpha + \beta} . \quad (8)$$

Here  $x_\infty$  denotes the steady stationary state fraction of open gates and  $\tau$  the relaxation time. At equilibrium, the probability for a channel to be in the open state is  $x_\infty$ , and the probability to be in the closed state is  $(1 - x_\infty)$ . The ratio of these two probabilities is given by the Boltzmann distribution,

$$\frac{x_\infty}{1 - x_\infty} = \exp\left(\frac{\Delta G_x}{kT}\right) , \quad (9)$$

where  $T$  is the absolute temperature,  $k$  is Boltzmann's constant, and  $\Delta G_x$  denote the energy difference between the open and the closed positions. Thus,

$$x_\infty = \left(1 + \exp\left[-\frac{\Delta G_x}{kT}\right]\right)^{-1} . \quad (10)$$

At equilibrium, each of the forward reactions must occur just as frequently as each of the reverse reactions, giving,

$$\frac{\alpha_i}{\beta_i} = \exp\left(\frac{\Delta G_x}{kT}\right) . \quad (11)$$

This is the principle of detailed balance which is present in dynamical systems (reversible mechanics). As in the standard model the rates are then assumed to be,

$$\alpha_i = \lambda_i \exp\left(\frac{\Delta G_x}{2kT}\right) \quad (12)$$

$$\beta_i = \lambda_i \exp\left(-\frac{\Delta G_x}{2kT}\right), \quad (13)$$

where  $\lambda_i$  is assumed to be independent of  $\Delta G_x$ . Thus the relaxation time (Eq. 8) can then be written as,

$$\tau = \frac{1}{\alpha + \beta} = \frac{1}{p_1 \alpha_1 + p_2 \alpha_2 + p_1 \beta_1 + p_2 \beta_2}. \quad (14)$$

Using Eqs. 4, 5, 12 and 13, we obtain

$$\tau = \frac{2 \cosh(\frac{\Delta G_b}{2kT})}{(\alpha_1 + \beta_1) \exp(-\frac{\Delta G_b}{2kT}) + (\alpha_2 + \beta_2) \exp(\frac{\Delta G_b}{2kT})} \quad (15)$$

$$= \frac{\cosh(\frac{\Delta G_b}{2kT})}{\cosh(\frac{\Delta G_x}{2kT}) [\lambda_1 \exp(-\frac{\Delta G_b}{2kT}) + \lambda_2 \exp(\frac{\Delta G_b}{2kT})]} \quad (16)$$

$$= \frac{\cosh(\frac{\Delta G_b}{2kT})}{\lambda \cosh(\frac{\Delta G_x}{2kT}) [\exp(-\frac{\Delta G_b}{2kT} - \gamma) + \exp(\frac{\Delta G_b}{2kT} + \gamma)]} \quad (17)$$

$$= \frac{\cosh(\frac{\Delta G_b}{2kT})}{2\lambda \cosh(\frac{\Delta G_x}{2kT}) \cosh(\frac{\Delta G_b}{2kT} + \gamma)}, \quad (18)$$

where,

$$\gamma = \frac{1}{2} \log\left(\frac{\lambda_2}{\lambda_1}\right) \quad (19)$$

$$\lambda = \sqrt{\lambda_1 \lambda_2}. \quad (20)$$

To be more specific the voltage dependences of  $\Delta G_x$  and  $\Delta G_b$  are needed. For the energy difference between the open state and the closed state we assume as usual,

$$\Delta G_x = G_{\text{closed}} - G_{\text{open}} \equiv q_x(v - v_x) - s_x T, \quad (21)$$

where the term  $q_x v_x$  is due to the difference in mechanical conformation energy between the two states;  $q_x v$  represents the electrical potential energy change associated with the redistribution of charge during the transition, and  $s_x$  is due to the difference in entropy between the two states. A similar expression can be assumed for the energy difference between the two barriers in route 1 and 2,

$$\Delta G_b = G_1 - G_2 \equiv q_b(v - v_b) - s_b T. \quad (22)$$

Here  $v$  is voltage, while  $q_x$ ,  $v_x$ ,  $s_x$ ,  $q_b$ ,  $v_b$ , and  $s_b$  are constants. One notes that the curve for the relaxation time  $\tau$  has a shift in position due to the term  $\gamma$ . Inserted for the special case  $\Delta G_b = \Delta G_x$  the above yields,

$$x_\infty = \left( 1 + \exp \left[ \frac{-q_x(v - v_x) + s_x T}{kT} \right] \right)^{-1} \quad (23)$$

$$\tau = \left( 2\lambda \cosh \left[ \frac{-q_x(v - v_x) + T(s_x - 2\gamma k)}{2kT} \right] \right)^{-1}. \quad (24)$$

Here we find that the voltage dependence of the curve for the relaxation time (Eq. 24) is shifted by an amount  $2\gamma kT/q_x$  relative to the steady state activation curve (Eq. 23), which means that the magnitude of the shift depends upon temperature. With  $\Delta G_b \neq \Delta G_x$  expression (Eq. 24) becomes more complex as follows from (Eq. 18), and the shape of the former curve is modified. This however, is dealt with in the next section.

## RESULTS

We now will compare the model with the experimental results of Clay *et al.* (1995) and show that it is consistent with the latter. Thus it presents a mechanism that represents a possible solution to the gating current paradox. The temperature-dependence of the currents were not considered in those experiments, so here  $s_x$  and  $s_b$  can be incorporated into  $v_x$  and  $v_b$ . With use of Eqs. 21 and 22, Eqs. 10 and 18 become,

$$x_\infty = \frac{1}{1 + \exp \left( \frac{v_x - v}{k_x} \right)} \quad (25)$$

$$\tau = \frac{\cosh(\frac{v - v_b}{2k_b})}{2\lambda \cosh(\frac{v - v_x}{2k_x}) \cosh(\frac{v - v_b}{2k_b} + \gamma)}, \quad (26)$$

where  $k_x = kT/q_x$  and  $k_b = kT/q_b$ . These expressions were evaluated numerically adjusting the parameters present to obtain a best possible fit to the experimental data. A least squares fit weighting various points in accordance with experimental uncertainty was used. The results of this evaluation is shown in the figure below where the data of Clay *et al.* (1995) is presented together with the curves given by Eqs. 25 and 26 using the parameters shown in the figure text.

However, the curves are not very sensitive to the values of these parameters except  $\gamma$ , so they can be varied quite a bit and still give essentially the same curves. From these curves we find that the model is fully consistent with the experimental results within the uncertainties in the latter.

## DISCUSSION

We have presented a Markov model that yields a possible solution to the gating current paradox announced by Clay *et al.* (1995). It gives a simple explanation of the voltage-shift of the bell-shaped curve for the relaxation time relative to the steady state activation curve. Also the width and shape of the relaxation time curve can be modified in a way consistent with experiments. A novel feature of the present model is that the voltage-shift is temperature dependent. It is not clear whether such a temperature dependence can be observed experimentally.

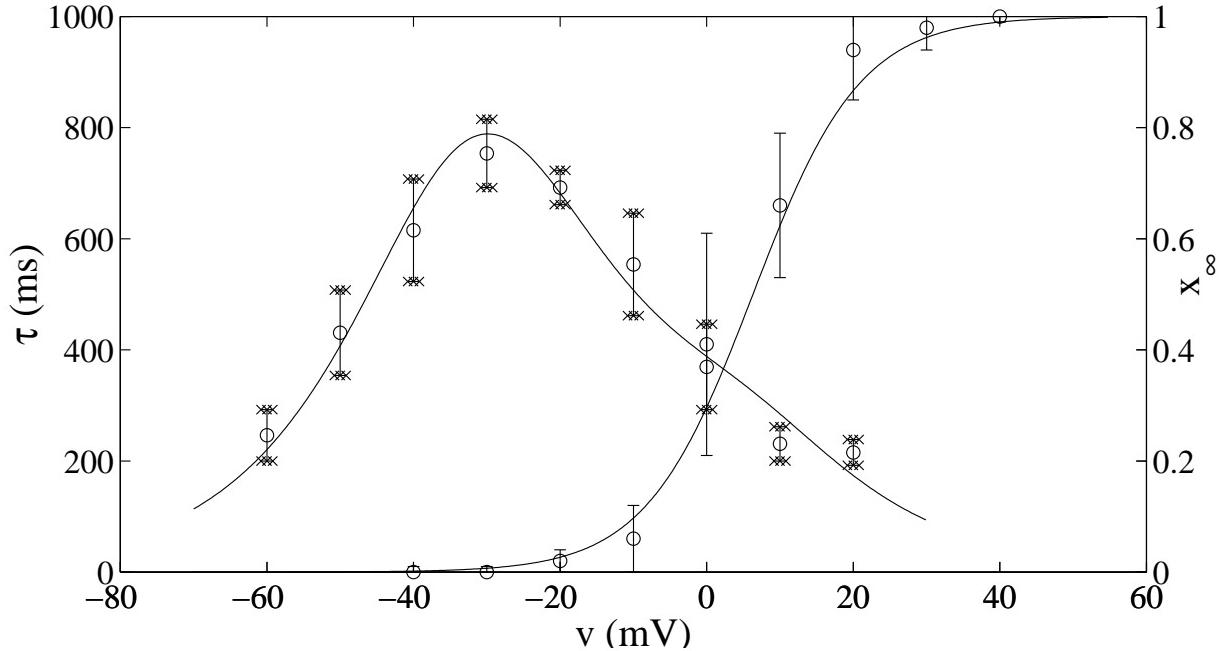


FIGURE The steady-state activation curve (Eq. 25) and the bell-shaped curve for the relaxation time (Eq. 26), with the parameters  $v_x = 6.31$  mV,  $k_x = 7.31$  mV,  $\lambda = 0.31 \text{ s}^{-1}$ ,  $v_b = -1.79$  mV,  $k_b = 7.99$  mV and  $\gamma = 1.89$ . The error bars indicate the  $\pm$  mean standard deviation from the six experiments of Clay *et al.* (1995).

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